Problem 6.

Calamity Jane goes to the bank to make a withdrawal, and is equally likely to find 0 or 1 customers ahead of her. The service time of the customer ahead, if present, is exponentially distributed with parameter >.. What is the CDF of Jane's waiting time?

Solution to Problem 3.6.

Let X be the waiting time and Y be the number of customers found. For x < 0, we have FX(x) = 0, while for x ≥ 0, FX(x) = P(X ≤ x) = 1 2 P(X ≤ x | Y = 0) + 1 2 P(X ≤ x | Y = 1). Since P(X ≤ x | Y = 0) = 1, P(X ≤ x | Y = 1) = 1 − e −λx , we obtain FX(x) = ( 1 2 (2 − e −λx), if x ≥ 0, 0, otherwise. Note that the CDF has a discontinuity at x = 0. The random variable X is neither discrete nor continuous

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In the box, there are two kinds of batteries, Type A and Type B. Their lifetime duration are exponential random variable with parameter, λA and λB. If the probability of taking Type A from the box is pA and Type B is pB­ (pA + pB = 1). Now knowing a battery has already used t hours, find the probability that it still can work for another s hours.

Sol>

X denotes the lifetime of the battery.

P{X>s+t|X>t} =

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=

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A point Y is selected at random from the interval [0, n]. Another point X is selected at random from the interval [Y, n]. Find the probability density function of X.

Answer. Let f(x,y) be the joint probability density function of X and Y . Clearly,

f(x,y) = fX|Y (x|y)fY (y). Thus fX(x) = .

Now fY (y) =1 /n,0 ≤ y ≤ n; 0 otherwise .and

fX|Y (x|y) = if 0 ≤ x ≤ y ≤ n; 0 otherwise

fXY(x,y) = fX|Y (x|y)\*fY(y) = if 0 ≤ x ≤ y ≤ n; 0 otherwise

Therefore, for 0 ≤ x ≤ n, fX(x) = = (ln(n−x) - ln(n))and hence

fX(x) = (ln(n−x) - ln(n)), 0 ≤ x ≤ n; 0 otherwise

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Q:

Let X and Y be two independent random variables with PDF

𝑓X(𝑥)=, 𝑥≥0, and 0, x<0;

𝑓Y(𝑦)=, 𝑦≥0, and 0, y<0.

Find PDF of Z=X+Y

A:

The joint distribution of 𝑓X 、𝑓Y  is 𝑓X,Y (x,y)=, x,y0, and 0, othetwise.

Now, transform(x,y)(z,x)

z=x+y y=z-x.

Since y0 z-x0 zx0

𝑓Z,X (z,x)=x(z-x) , zx0 ;and 0, othetwise.

𝑓Z (z)=

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1.

Suppose an engineer has designed a probe, its probability of locating its target is a random variable T with PDF

The random variable T is the time variable(seconds as units), meaning the probe only has at most 1 second to locate its target. In essence, the probe will have a fixed probability T = t of locating its target under specific conditions. Each search is an independent event.

(Hint: )

(a) Find the probability that the probe locates its target on the first search.

(b) Given that the probe has located its target on its first search, find the conditional PDF of T.

Solution

1. Let A be the event that the first search was successful. To calculate the probability P(A), we use the continuous version of the total probability theorem:

Therefore,

2. Using Bayes rule,